

Interlacing

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The blue text are links to the main site. To see the detail of the interlacing, zoom in.

1 Introduction

Interlacing takes many forms on this site. The most frequent form is narrow white bands that go under and over other bands in order. For a simple one see [Alhambra, Salon de Comares](#) and for a complex one see [Seville, Mosaic of the large Court](#).

A study of the interlace patterns has been undertaken in [1]. Grünbaum and Shephard provides an analysis with several examples. Here, we use computer software to illustrate the interlace patterns rather than undertaking an analysis as such.

A completely different method of presenting such patterns is with each individual interlace coloured, see [Tilings with coloured interlacing](#). Note that [Saadian Tombs, Marrakech](#) is special since the individual interlaces change colour.

2 Path tracing

Now consider [Royal Palace Throne Room, Meknes](#). It does not have the traditional narrow band of up/down interlacing, but this *could* have been added to this design. Our analysis of interlacing considers the designs themselves rather than how they are presented — hence designs such as this one are included.

A pattern has interlaces if an even number of lines join at each vertex, and the lines go straight across the vertex when the number of lines is four or more. One difficulty with the patterns which allows interlacing is to determine how many interlaces there are, both finite and infinite. A program has been produced to facilitate the counting of the interlaces which colours one example of each.

A lot of patterns have very simple interlace patterns — in those cases, the interlacing colouring program has not been used and those patterns are not shown here. Also, the program would have technical difficulties in analysing very complex patterns, and does not work at all with 6-way cross overs — hence these are excluded.

In examining these interlace displays, a question arises. Take a single interlace which crosses itself. For each of those crossings, mark the up/down property as it appears in the full pattern. Does the single interlace display the same up/down behaviour? I have not been able to find a case which does not display the up/down behaviour.

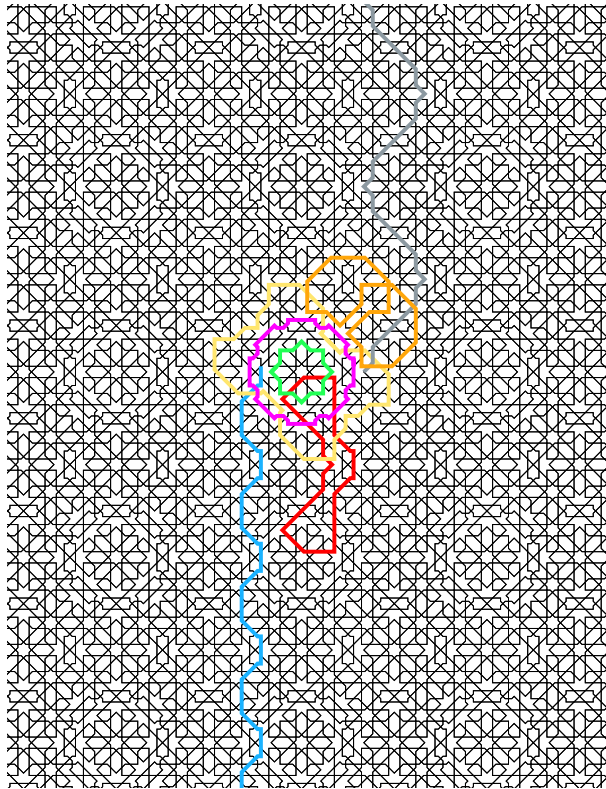


Figure 1: [Royal Palace Throne Room, Meknes](#)

There are a number of points to note about this interlace pattern:

1. The thin black lines are members of one of the seven interlaces but only one instance is shown (to avoid a confusion of colour).
2. The green and purple interlaces have 4-fold symmetry (round the centre of symmetry).
3. The yellow interlace has only two-fold symmetry which implies there is another yellow interlace round the centre of symmetry.
4. The bow-tile red interlace has a line of reflective symmetry round its centre.
5. There are nine times as many tiles shown here than on the original pattern which can be seen by clicking on the caption.

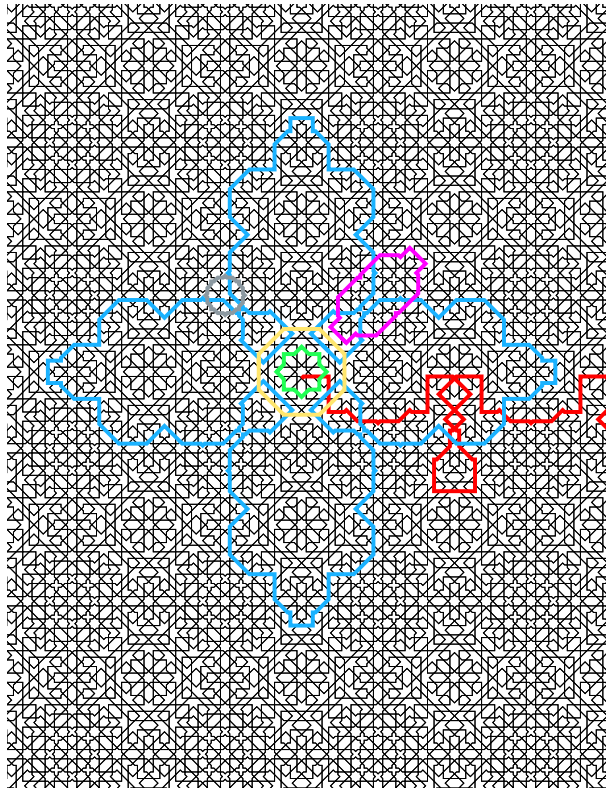


Figure 2: Royal Palace Throne Room, Meknes

There are a number of points to note about this interlace pattern:

1. To get the interlace within the printed area, 16 times as many tiles were needed compared with the original pattern.
2. The red interlace is clearly infinite, but this is less easy to see since part of it goes outside the printed area so the program omits to notice that it comes back again!
3. Problems arise if the pattern has to be much bigger than this to show the interlacing properly.

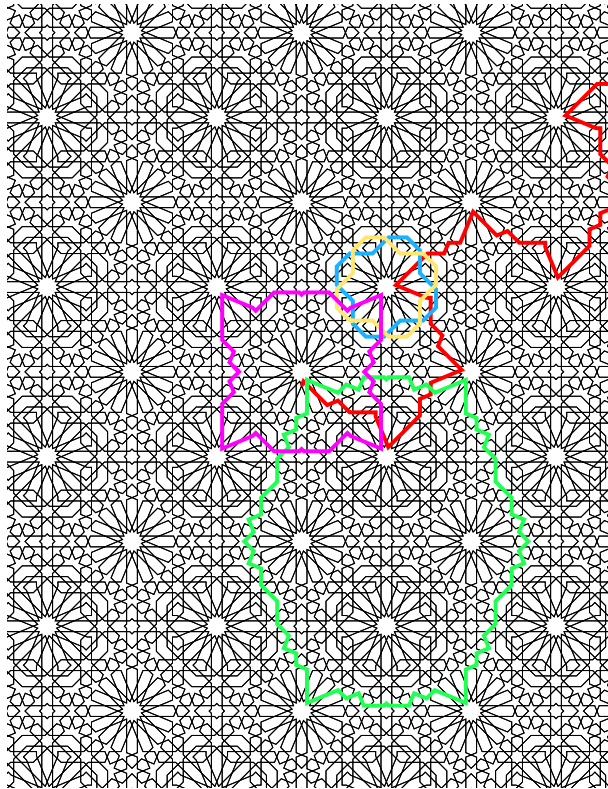


Figure 3: Sa'di tombs, Marrakech

There are a number of points to note about this interlace pattern:

1. The blue and yellow interlaces are really the same one. The program does not handle reflections and these two are reflections of each one (along the diagonal).
2. The green interlace fits into the printed area, but clearly some finite interlaces may not.

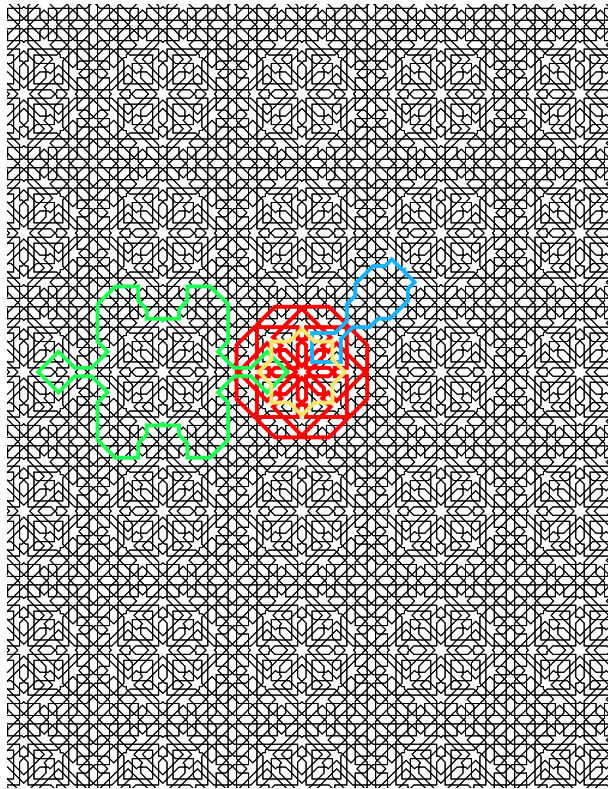


Figure 4: [Telouet, Royal Residence](#)

There is one point to note about this interlace pattern:

1. The interlace pattern has 10,000 polygons. Unlikely to be able to produce more complex patterns than this.

Further examples of path tracing is to be found in [2].

3 Interlacing issues

When a pattern has interlacing, then strictly the pattern's symmetry does not include reflections. For the point of view of this web site, this is unsatisfactory, since we regard the way lines are drawn, including the use of interlacing. For this reason, when a pattern has interlacing, we ignore this in determining the symmetry.

To allow interlacing, a pattern is usually two-colourable. Moreover, the four-way intersections are usually straight. There are a few exceptions to this on the web site, for instance [Alhambra, Sala de la Barca](#).

References

- [1] B. Grünbaum and G. C. Shephard, *Interlace Patterns in Islamic and Moorish Art*. Leonardo, Vol. 25, No. 3/4, pp. 331-339. 1992.
- [2] Brian Wichmann and David Wade. *Islamic Design: a Mathematical Approach*. Springer. 2018. ISBN 978-3-319-699

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